Student Workbook

Unit Review — Advanced Trig, Polar, Parametric & Vectors

Name: ______________________ Date: ______________ Class: ________

UNIT SUMMARY

Consolidate your understanding of the Law of Sines and Cosines, polar coordinates and graphs, complex numbers in polar form, parametric equations and curves, and working with vectors.

Formula Reference

$$a/\sin A = b/\sin B = c/\sin C \mid c^2 = a^2 + b^2 - 2ab\cos C$$

Law of Sines	Area=½ab sinC. SSA ambiguous: h=b sinA. h>a: none; h=a: 1 right △; h<a<b: 2 △; a≥b: 1 △
Law of Cosines	$\cos C=(a^2+b^2-c^2)/(2ab)$. Heron's: $s=(a+b+c)/2$, Area=$\sqrt{(s(s-a)(s-b)(s-c))}$
Polar	$x=r\cos\theta$, $y=r\sin\theta$. $r^2=x^2+y^2$, $\tan\theta=y/x$. $(r,\theta)=(r,\theta+2\pi n)=(-r,\theta+\pi+2\pi n)$
Polar Curves	Limaçon $r=a\pm b\cos\theta$ (cardioid if a=b). Rose $r=a\cos(n\theta)$: n petals if odd, 2n if even
Complex Polar	$z=r \text{ cis } \theta$. Multiply: mult. moduli, add args. De Moivre's: $[r \text{ cis } \theta]^n=r^n \text{ cis}(n\theta)$
Parametric	$x=f(t)$, $y=g(t)$. Projectile: $x=v_0\cos\alpha\cdot t$, $y=v_0\sin\alpha\cdot t-½gt^2$
Vectors	$\lvert v\rvert=\sqrt{(a^2+b^2)}$. $u\cdot v=ac+bd$. $\cos\theta=(u\cdot v)/(\lvert u\rvert\lvert v\rvert)$. Orthogonal iff $u\cdot v=0$

Review Worked Examples

EXAMPLE 1

Law of Sines — SSA Ambiguous Case: A=35°, a=9, b=14. Find all possible triangles

- h = b sinA = 14 sin35° ≈ 8.03. Since h < a < b (8.03<9<14), there are two solutions
- sinB = b sinA/a = 14 sin35°/9 ≈ 0.892 → B_1≈63.2°, B_2=180°−63.2°≈116.8°
- Triangle 1: C_1≈81.8°, c_1=9 sin81.8°/sin35°≈15.5. Triangle 2: C_2≈28.2°, c_2≈7.4

Answer: Two triangles: (B≈63.2°,C≈81.8°,c≈15.5) and (B≈116.8°,C≈28.2°,c≈7.4)

EXAMPLE 2

Law of Cosines — SSS: a=8, b=11, c=15. Find all angles

- $\cos C=(a^2+b^2-c^2)/(2ab)$=(64+121−225)/176=−40/176≈−0.227; C≈103.1°
- $\cos B=(a^2+c^2-b^2)/(2ac)$=(64+225−121)/240=168/240=0.7; B≈45.6°
- A=180°−103.1°−45.6°≈31.3°

Answer: A≈31.3°, B≈45.6°, C≈103.1°

EXAMPLE 3

Polar/Complex: Multiply z_1=3 cis(π/4) and z_2=2 cis(π/3), then find z_1^3 using De Moivre's

- Product: multiply moduli → 3·2=6; add arguments → π/4+π/3=7π/12
- $z_1 z_2$ = 6 cis(7π/12)
- De Moivre's: $z_1^3 = 3^3$ cis(3·π/4) = 27 cis(3π/4)

Answer: $z_1 z_2$ = 6 cis(7π/12); z_1^3 = 27 cis(3π/4)

EXAMPLE 4

Parametric Projectile: A ball is launched at 30 m/s at 50°. Find max height, range, time of flight

- v_x=30cos50°≈19.3 m/s; v_y=30sin50°≈23.0 m/s
- $t_{max}=v_y/g$≈2.35 s; $y_{max}=v_y \cdot t_{max} - ½g \cdot t_{max}^2$≈54.1−27.0≈27.0 m
- $t_{flight}=2 \cdot t_{max}$≈4.69 s; Range=$v_x \cdot t_{flight}$≈19.3·4.69≈90.5 m

Answer: Max height ≈ 27.0 m, Range ≈ 90.5 m, Time of flight ≈ 4.69 s

EXAMPLE 5

Vectors: u=⟨3,−4⟩, v=⟨5,12⟩. Find u·v, the angle between them, and whether orthogonal

- u·v = 3(5)+(−4)(12) = 15−48 = −33
- |u|=√(9+16)=5; |v|=√(25+144)=13
- cosθ=(u·v)/(|u||v|)=−33/65; θ=arccos(−33/65)≈120.5°

Answer: u·v = −33, θ ≈ 120.5°, not orthogonal (since u·v ≠ 0)

Common Mistakes to Avoid

✗ Using Law of Sines for SAS or SSS.	✓ SAS and SSS require the Law of Cosines. Law of Sines needs an angle-side opposite pair.
✗ Forgetting the second triangle in the SSA ambiguous case.	✓ Always check if h < a < b (where h=b sinA). If so, there are two valid triangles.
✗ Confusing polar point (r,θ) with rectangular (x,y).	✓ Always convert using x=r cosθ, y=r sinθ before plotting in rectangular.
✗ Stopping at k=0 when finding nth roots of a complex number.	✓ There are n distinct nth roots. Use k=0,1,…,n−1 in $r^{1/n}$ cis((θ+2πk)/n).

✗ Plotting (t,x) or (t,y) instead of (x,y) for parametric curves.

✓ Parametric curves are plotted in the xy-plane. t is used only to generate (x,y) pairs.

✗ Computing |u+v| = |u|+|v|.

✓ Find u+v first, then compute its magnitude. Triangle inequality: |u+v| ≤ |u|+|v|.

Math Tips

★ Decision rule for triangles: AAS/ASA → Law of Sines; SAS/SSS → Law of Cosines; SSA → Law of Sines (check ambiguous case).

★ Polar form makes complex multiplication elegant: multiply moduli, add arguments. Division: divide moduli, subtract arguments.

★ For parametric projectile motion, horizontal and vertical components are independent: horizontal is constant velocity, vertical is constant acceleration.

★ The dot product u·v=0 is the fastest test for perpendicularity — no need to find the angle.

★ Rose curves: r=a cos(nθ) has n petals if n is odd, 2n petals if n is even.

★ Polar coordinates have infinitely many representations: (r,θ)=(r,θ+2πn)=(−r,θ+π+2πn).